

Interaction of Moving Branes with Background Massless and Tachyon Fields in Superstring Theory

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Abstract

Using the boundary state formalism we study a moving Dp -brane in a partially compact spacetime in the presence of the background fields: Kalb-Ramond $B_{\mu\nu}$, a $U(1)$ gauge field A_α and the tachyon field. The boundary state enables us to obtain interaction amplitude of two branes with above background fields. The branes are parallel or perpendicular to each other. Presence of the background fields, compactification of some directions of the spacetime, motion of the branes and arbitrariness of the branes' dimensions give a general feature to the system. Due to the tachyon fields and velocities of the branes, the behavior of the interaction amplitude reveals obvious differences from what is conventional.

PACS numbers: 11.25.-w; 11.25.Mj

Keywords: Moving D-branes; Background fields; Boundary state; Interaction.

1 Introduction

The discovery of the D-branes, as an inevitable part of the string theory [1], induced to study the properties and interactions of the branes. One of the most applicable methods for this purpose, is the boundary state formalism. Boundary state is a BRST invariant state which describes the creation of closed string from vacuum.

Among achievements in this formalism is extending it to the superstring theory and considering the contribution of the conformal and super conformal ghosts to the boundary state [2]. There are separate studies which add background fields such as the Kalb-Ramond field $B_{\mu\nu}$, $U(1)$ gauge field in the compact spacetime [3] and the tachyon field [4, 5, 6] to the subject of boundary state. These background fields give a more general feature to the subject. Apart from the longitudinal fluctuations of the brane (for instance the $U(1)$ gauge field and tachyon field), transverse fluctuations of the brane [7] should also be considered. This enables us to interpret it as a dynamical object. This can be performed by considering velocity for the brane [8, 9]. These motivated us to take into account all background fields and also compactification of some directions of the spacetime to study moving branes in a general framework of superstring theory. This general set-up can not be found in the literatures of the boundary state and branes' interaction.

Since open strings are quantum excitations of brane [10], presence of the open string tachyon reveals the instability of the brane. In the bosonic string theory this is a natural property, while in the superstring theories this occurs in special cases. For instance there are Dp -branes with wrong dimensions in the type IIA and type IIB superstring theories; That is, there are Dp -branes with odd dimensions in the type IIA theory and even dimensions in the type IIB theory! [11] which are unstable. Actually this instability can be removed by rolling of the tachyon toward its minimum potential [12]. During this process tachyon energy dissipates to the bulk modes and an unstable system reach to a stable state which consists of lower dimensional branes or just the closed string vacuum without any D-brane [10]. Usually in the literature the tachyon field has been considered in just one dimension and its effects have been studied on a space-filling brane, while in the present paper we consider a Dp -brane with arbitrary dimension, and hence the tachyon field possesses components along all directions of the brane worldvolume.

In this manuscript we calculate the boundary state corresponding to a moving Dp -brane in the presence of the background fields $B_{\mu\nu}$, $U(1)$ gauge field and tachyon. Then we use this boundary state to detect the interaction between two moving D-branes. There is no restriction on the branes' dimensions and they can be parallel or perpendicular to each

other. To keep the generality, we let some of the spacetime directions be compact. We will observe that presence of the tachyon prevents the closed string from wrapping around the compact directions. Using the boundary state, we calculate the interaction amplitude between two branes in the NS-NS and the R-R sectors. Due to the presence of the velocities and the background tachyon fields there is no cancellation between these amplitudes. This occurs even for the similar and parallel Dp -branes with the same background fields. We shall observe that the interaction amplitude vanishes after a long time (or equivalently for large distances of branes). The origin of this effect is the rolling of the background tachyon field and decaying of the D-branes in this limit.

So putting all these together allows us to study a system in the most general feature to obtain considerable results in spite of some mathematical difficulties because of considering longitudinal and transverse fluctuations simultaneously.

2 The boundary state associated with a Dp -brane

To obtain the boundary state corresponding to a moving brane in the presence of the anti-symmetric field $B_{\mu\nu}$ in bulk, and tachyon and $U(1)$ gauge fields on the boundary we consider the following sigma-model action for closed string

$$S = -\frac{1}{4\pi\alpha'} \int_{\Sigma} d^2\sigma (\sqrt{-g} g^{ab} G_{\mu\nu} \partial_a X^\mu \partial_b X^\nu + \varepsilon^{ab} B_{\mu\nu} \partial_a X^\mu \partial_b X^\nu) + \frac{1}{2\pi\alpha'} \int_{\partial\Sigma} d\sigma \left(A_\alpha \partial_\sigma X^\alpha + V^i X^0 \partial_\tau X^i + \frac{1}{2} U_{\alpha\beta} X^\alpha X^\beta \right), \quad (1)$$

where the first integral is on the worldsheet of the closed string, exchanged between the branes, and the second one is on the boundary of this worldsheet which can be at $\tau = 0$ or $\tau = \tau_0$. The $U(1)$ gauge field A_α , lives on the Dp -brane worldvolume and V^i is the brane velocity component along X^i -direction. The set $\{X^\alpha\}$ and $\{X^i\}$ specify the directions along and perpendicular to the Dp -brane worldvolume, respectively. The term $\frac{1}{2} U_{\alpha\beta} X^\alpha X^\beta$ with constant symmetric matrix $U_{\alpha\beta}$ specifies the tachyon profile. According to [13] the tachyon field appears in a square form in the action to produce a Gaussian integral. We take the tachyon field to have components along the Dp -brane worldvolume. Here we consider $G_{\mu\nu}$ as the flat spacetime metric with the signature $\eta_{\mu\nu} = \text{diag}(-1, 1, \dots, 1)$ and the Kalb-Ramond field $B_{\mu\nu}$ to be a constant field.

Vanishing the variation of the action (1) with respect to $X^\mu(\sigma, \tau)$, gives us the equations of motion as well as the boundary equations of the emitted (absorbed) closed string.

2.1 Bosonic part of the boundary state

Boundary equations resulted from the action (1) at $\tau = 0$ are as in the following

$$\begin{aligned} [\partial_\tau(X^0 - V^i X^i) + \mathcal{F}^0_\alpha \partial_\sigma X^\alpha - U^0_\alpha X^\alpha]|B_x, \tau = 0\rangle &= 0, \\ (\partial_\tau X^{\bar{\alpha}} + \mathcal{F}^{\bar{\alpha}}_\beta \partial_\sigma X^\beta - U^{\bar{\alpha}}_\beta X^\beta)|B_x, \tau = 0\rangle &= 0, \\ (X^i - V^i X^0 - y^i)|B_x, \tau = 0\rangle &= 0. \end{aligned} \quad (2)$$

In above boundary conditions $X^{\bar{\alpha}}$ shows the spatial directions of the brane worldvolume (i.e. $\bar{\alpha} \neq 0$) and \mathcal{F} is the total field strength, $\mathcal{F}_{\alpha\beta} = \partial_\alpha A_\beta - \partial_\beta A_\alpha - B_{\alpha\beta}$, which contains B field as well as the $U(1)$ gauge field. Note that we have assumed the mixed elements of the Kalb-Ramond field to be zero, i.e. $B^\alpha_i = 0$.

The solution of the closed string equation of motion is

$$X^\mu(\sigma, \tau) = x^\mu + 2\alpha' p^\mu \tau + 2L^\mu \sigma + \frac{i}{2}\sqrt{2\alpha'} \sum_{m \neq 0} \frac{1}{m} (\alpha_m^\mu e^{-2im(\tau-\sigma)} + \tilde{\alpha}_m^\mu e^{-2im(\tau+\sigma)}). \quad (3)$$

L^μ is zero for non-compact directions, and $L^\mu = N^\mu R^\mu$ for the compact direction X^μ with the compactification radius R^μ and the closed string winding number N^μ . The closed string center of mass momentum is $p^\mu = \frac{M^\mu}{R^\mu}$, where M^μ is the momentum number of it. Substituting this solution into the boundary equations (2), gives them in terms of oscillators and zero modes. During this process an interesting condition on the closed string winding is obtained

$$U^\alpha_{\bar{\beta}} L_{\text{op}}^{\bar{\beta}} |B_x, \tau = 0\rangle = 0.$$

We assumed that there is no compactification along time direction and hence $L^0 = 0$. In the case of invertibility of the matrix $U^\alpha_{\bar{\beta}}$, this equation reduces to $L_{\text{op}}^{\bar{\alpha}} |B_x, \tau = 0\rangle = 0$. Therefore, the presence of the background tachyon field prevents closed string from wrapping around compact directions which are parallel to the brane worldvolume.

Utilizing the coherent state method [14] to solve the boundary equations (2) for oscillating modes leads to the following state

$$|B_{\text{osc}}, \tau = 0\rangle = \prod_{n=1}^{\infty} [\det M_{(n)}]^{-1} \exp \left[- \sum_{m=1}^{\infty} \left(\frac{1}{m} \alpha_{-m}^\mu \mathcal{S}_{(m)\mu\nu} \tilde{\alpha}_{-m}^\nu \right) \right] |0\rangle, \quad (4)$$

where the matrix $\mathcal{S}_{(m)}$ is defined by

$$\begin{aligned} \mathcal{S}_{(m)} &= S_{(m)} + (S_{(-m)}^{-1})^T, \\ S_{(m)} &= M_{(m)}^{-1} N_{(m)}. \end{aligned} \quad (5)$$

The matrices $M_{(m)}$ and $N_{(m)}$ which are functions of background fields are defined by

$$M_{(m)}^\mu{}_\nu = \Omega^\mu{}_\nu - \frac{i}{2m} U^\alpha{}_\beta \delta^\mu{}_\alpha \delta^\beta{}_\nu \quad (6)$$

where

$$\left\{ \begin{array}{l} \Omega^0{}_\mu = \delta^0{}_\mu - V^i \delta^i{}_\mu - \mathcal{F}^0{}_\alpha \delta^\alpha{}_\mu, \\ \Omega^{\bar{\alpha}}{}_\mu = \delta^{\bar{\alpha}}{}_\mu - \mathcal{F}^{\bar{\alpha}}{}_\beta \delta^\beta{}_\mu, \\ \Omega^i{}_\mu = \delta^i{}_\mu - V^i \delta^0{}_\mu, \end{array} \right. \quad (7)$$

and

$$\left\{ \begin{array}{l} N_{(m)}^0{}_\mu = \delta^0{}_\mu - V^i \delta^i{}_\mu + \mathcal{F}^0{}_\alpha \delta^\alpha{}_\mu + \frac{i}{2m} U^0{}_\alpha \delta^\alpha{}_\mu, \\ N_{(m)}^{\bar{\alpha}}{}_\mu = \delta^{\bar{\alpha}}{}_\mu + \mathcal{F}^{\bar{\alpha}}{}_\beta \delta^\beta{}_\mu + \frac{i}{2m} U^{\bar{\alpha}}{}_\beta \delta^\beta{}_\mu, \\ N_{(m)}^i{}_\mu = -\delta^i{}_\mu + V^i \delta^0{}_\mu. \end{array} \right. \quad (8)$$

When we solve the boundary equations, the matrix $(S_{(-m)}^{-1})^T$ also appears in the Eq. (5). This is due to the fact that the matrix $S_{(m)}$ is mode-dependent and generally is not orthogonal. In the absence of the tachyon field, S becomes mode-independent and orthogonal, so $\mathcal{S} = S$ [3]. The infinite product in the Eq. (4), which comes from path integral, can be regularized [15] as

$$\prod_{n=1}^{\infty} [\det M_{(n)}]^{-1} = \sqrt{\det \Omega} \det \left[\Gamma \left(\frac{U}{1 + 2i\Omega} \right) \right]. \quad (9)$$

From now on we consider a selected direction X^{i_0} for the motion of the Dp -brane and hence the other components of the velocity are zero. We also define $V^{i_0} = V$. By this assumption, the zero mode part of the boundary state becomes

$$\begin{aligned} |B_x, \tau = 0\rangle^{(0)} = & \frac{T_p}{2} \int_{-\infty}^{\infty} \prod_{\alpha} dp^\alpha \left\{ \exp \left[-4i\alpha' (U^{-1})_{\alpha\beta} \left(\left(1 - \frac{1}{2} \delta_{\alpha\beta}\right) p^\alpha p^\beta + V p^{i_0} p^\beta \delta_0^\alpha \right) \right] \right. \\ & \times \delta(x^{i_0} - Vx^0 - y^{i_0}) \prod_{i' \neq i_0} \delta(x^{i'} - y^{i'}) \\ & \left. \times \prod_{\alpha} |p_L^\alpha = p_R^\alpha\rangle \prod_{i' \neq i_0} |p_L^{i'} = p_R^{i'} = 0\rangle |p_L^{i_0} = p_R^{i_0} = \frac{1}{2} V p^0\rangle \right\}. \end{aligned} \quad (10)$$

Two delta functions indicate the position of the brane along the perpendicular directions. The integration over the momenta indicates that the effects of all values of the momentum components have been taken into account. In addition, the equality $p_L^\alpha = p_R^\alpha$ originates from the un-wrapping of the closed string around the brane directions and non-compactness of the time direction.

There are two special limiting cases for U . In the limit $U_{\alpha\beta} \rightarrow 0$, the oscillating part of the boundary state, i.e. the Eq. (4), reduces to a boundary state corresponding to a moving Dp -brane in the absence of tachyon field, [9].

When we send some of the elements of U to infinity, somehow we are looking at the boundary state in the concept of tachyon condensation. This condensation can be performed on some or all elements of the tachyon matrix U . Without loss of generality consider U as a diagonal matrix. By sending to infinity a spatial element $U_{\bar{\alpha}\alpha}$, the boundary state transforms to the one related to a moving $D(p-1)$ -brane which has lost its dimension along the $X^{\bar{\alpha}}$ -direction, and is in the presence of a new tachyon field $U'_{(p-1)\times(p-1)}$ which does not include the component $U_{\bar{\alpha}\alpha}$.

Notable point here is that although in the process of condensation along the $X^{\bar{\alpha}}$ -direction the matrices M and \mathcal{S} in boundary state (4) change to lower dimensional ones, as expected, the effect of condensed component remains as a $\sqrt{U_{\bar{\alpha}\alpha}}$ factor after regularization of infinite product $\prod_{n=1}^{\infty} [\det M_{(n)}]^{-1}$. This result is different from the conventional case in which this factor is canceled by the factor $\frac{1}{\sqrt{U_{\bar{\alpha}\alpha}}}$ from zero mode part, that is absent here.

When condensation occurs along the time component of the tachyon matrix, $U_{00} \rightarrow \infty$, beside the decreasing of the brane worldvolume dimension in the X^0 -direction, the brane loses its velocity, too. In other words, the tachyon condensation along the temporal direction fixes the Dp -brane in time and space, i.e. makes an instantonic Dp -brane which has no velocity.

2.2 Fermionic part of the boundary state

To find boundary equations for the fermionic degrees of freedom there are two ways: 1) supersymmetrizing the action (1) and putting the variation of the fermionic part of the action equal to zero; 2) Since the supersymmetrized action is invariant under the global worldsheet supersymmetry transformations, we can perform the worldsheet supersymmetry on the bosonic boundary Eqs. (2) and transform them to fermionic ones. Here we choose the second approach. So the fermionic boundary equations are

$$[-i\eta(\psi_+^0 - V^i\psi_+^i) + (\psi_-^0 - V^i\psi_-^i) + \mathcal{F}^0_{\alpha}(-i\eta\psi_+^{\alpha} - \psi_-^{\alpha}) - U^0_{\nu}(-i\eta\psi_+^{\nu} + \psi_-^{\nu})]|B_{\psi}, \eta, \tau = 0\rangle = 0,$$

$$[-i\eta\psi_+^{\bar{\alpha}} - \psi_-^{\bar{\alpha}} + \mathcal{F}^{\bar{\alpha}}_{\beta}(-i\eta\psi_+^{\beta} + \psi_-^{\beta}) - U^{\bar{\alpha}}_{\nu}(-i\eta\psi_+^{\nu} - \psi_-^{\nu})]|B_{\psi}, \eta, \tau = 0\rangle = 0,$$

$$[-i\eta(\psi_+^i - V^i\psi_+^0) - (\psi_-^i - V^i\psi_-^0)]|B_{\psi}, \eta, \tau = 0\rangle = 0. \quad (11)$$

With respect to the solution of the equations of motion for the fermions,

$$\psi_-^{\mu} = \sum_k \psi_k^{\mu} e^{-2ik(\tau-\sigma)} \quad , \quad \psi_+^{\mu} = \sum_k \tilde{\psi}_k^{\mu} e^{-2ik(\tau+\sigma)}, \quad (12)$$

the boundary state Eqs. (11) can be represented as

$$(\psi_k^\mu - i\eta S^\mu_{(k)\nu} \tilde{\psi}_{-k}^\nu) |B_\psi, \eta, \tau = 0\rangle = 0. \quad (13)$$

Note that in the Eqs. (12) and (13), k is an integer number m for the R-R sector, $\psi_m^\mu = d_m^\mu$ and $\tilde{\psi}_m^\mu = \tilde{d}_m^\mu$ while in the NS-NS sector, k is a half-integer number r with $\psi_r^\mu = b_r^\mu$ and $\tilde{\psi}_r^\mu = \tilde{b}_r^\mu$. The constant number η can be $+1$ or -1 . No matter we choose $+1$ or -1 , because for gaining the interaction of the branes, we need to use the boundary state which has been affected by the GSO projector. As will be seen, this projection operator causes the both states with $\eta = +1$ and $\eta = -1$ to contribute to the interaction.

Similar to the bosonic part we should also consider the portion of the super conformal ghosts in the fermionic boundary state. The super ghosts include the commuting fields β , γ , $\tilde{\beta}$ and $\tilde{\gamma}$.

2.2.1 The NS-NS Sector

According to the Eq. (13), the resultant NS-NS sector boundary state of the fermions is given by

$$|B_\psi, \eta, \tau = 0\rangle_{\text{NS}} = \prod_{r=1/2}^{\infty} [\det M_{(r)}] \exp \left[i\eta \sum_{r=1/2}^{\infty} (b_{-r}^\mu \mathcal{S}_{(r)\mu\nu} \tilde{b}_{-r}^\nu) \right] |0\rangle_{\text{NS}}. \quad (14)$$

When the path integral is computed the determinant will be reversed in comparing to the bosonic case, the Eq. (4). This is due to the Grassmann nature of the integration variables [2]. As r is half integer, regularization of this infinite product is

$$\prod_{r=1/2}^{\infty} [\det M_{(r)}] = \det \left(\frac{\sqrt{\pi}}{\Gamma[\frac{U}{2i\Omega} + \frac{1}{2}]} \right). \quad (15)$$

2.2.2 The R-R Sector

For acquiring the boundary state in the R-R sector, we have to follow the same procedure of the NS-NS sector with a bit difference which needs a careful notice. Since $k = m$ in the Eq. (13) runs over integers in the R-R sector, there is a zero mode which affects the boundary state. Solving the Eq. (13) in the R-R sector, yields the following boundary state

$$|B_\psi, \eta, \tau = 0\rangle_{\text{R}} = \prod_{m=1}^{\infty} [\det M_{(m)}] \exp \left[i\eta \sum_{m=1}^{\infty} (d_{-m}^\mu \mathcal{S}_{(m)\mu\nu} \tilde{d}_{-m}^\nu) \right] |B_\psi, \eta\rangle_{\text{R}}^{(0)}. \quad (16)$$

Since m is an integer number, regularization of the infinite product is exactly similar to the bosonic case (of course here the determinant is inverse of the bosonic case)

$$\prod_{m=1}^{\infty} [\det M_{(m)}] = \left\{ \sqrt{\det \Omega} \det \left(\Gamma \left[1 + \frac{U}{2i\Omega} \right] \right) \right\}^{-1}. \quad (17)$$

The state $|B_\psi, \eta\rangle_{\text{R}}^{(0)}$ in the Eq. (16) is the zero mode boundary state

$$|B_\psi, \eta\rangle_{\text{R}}^{(0)} = \left[C\Gamma_{11} \left(\frac{1 + i\eta\Gamma_{11}}{1 + i\eta} \right) \exp\left(\frac{1}{2}\Phi_{\mu\nu}\Gamma^\mu\Gamma^\nu\right) \right]^{AB} |A\rangle|\tilde{B}\rangle, \quad (18)$$

where $|A\rangle|\tilde{B}\rangle$ is the vacuum of the zero modes d_0^μ and \tilde{d}_0^μ . C is the charge conjugate matrix, and the antisymmetric matrix Φ is defined in terms of the matrix \mathcal{S} ,

$$\mathcal{S} = (1 - \Phi)^{-1}(1 + \Phi). \quad (19)$$

Details of obtaining the Eqs. (18) and (19) is shown in the appendix A. Since the matrix \mathcal{S} should be orthogonal $\mathcal{S}^{-1} = \mathcal{S}^T$, its definition $\mathcal{S}_{(m)} = S_{(m)} + [(S_{(-m)})^{-1}]^T$ implies that the matrix S should satisfy the following relation

$$S_{(m)}^T - S_{(m)}^{-1} = S_{(-m)}^T + S_{(-m)}^{-1}. \quad (20)$$

According to the Eqs. (5)-(8) S is defined in terms of the background fields. Thus, the Eq. (20) imposes a relation between these background fields. When in the action (1) the tachyon and velocity are put to zero, we receive $\Phi = \mathcal{F}$ and hence the term $\exp(\frac{1}{2}\Phi_{\alpha\beta}\Gamma^\alpha\Gamma^\beta)$ reduces to the known $\exp(\frac{1}{2}\mathcal{F}_{\alpha\beta}\Gamma^\alpha\Gamma^\beta)$ [3].

3 Interaction of the branes

Interaction amplitude between Dp_1 and Dp_2 -branes in each sector is defined by $\mathcal{A}_{\text{NS-NS,R-R}} = 2\alpha' \int_0^\infty dt \, {}_{\text{NS,R}}\langle B^1, \tau = 0 | e^{-tH_{\text{NS,R}}} | B^2, \tau = 0 \rangle_{\text{NS,R}}$. Total Hamiltonian $H_{\text{NS,R}}$ is sum of the Hamiltonians of X^μ 's, ψ^μ 's, ghosts and superghosts in each sector. For calculation of the interaction amplitude we need the total projected boundary state. The total boundary state of each sector is

$$|B, \eta, \tau = 0\rangle_{\text{NS,R}} = |B_X, \tau = 0\rangle |B_{\text{gh}}, \tau = 0\rangle |B_\psi, \eta, \tau = 0\rangle_{\text{NS,R}} |B_{\text{sgh}}, \eta, \tau = 0\rangle_{\text{NS,R}}.$$

In the appendix B the projection process is discussed. Thus, the total projected boundary states find the feature of the Eqs. (40) and (41).

3.1 Interaction amplitude in the NS-NS sector

Using the boundary state (40) for NS-NS sector, after a long calculation the total interaction amplitude in this sector is acquired as

$$\mathcal{A}_{\text{NS-NS}} = \frac{\alpha' V_{\overline{u}}}{8(2\pi)^{d_i}} \frac{T_{p_1} T_{p_2}}{|V_1 - V_2|} \prod_{m=1}^{\infty} \frac{\det[M_{(m-1/2)1} M_{(m-1/2)2}]}{\det[M_{(m)1} M_{(m)2}]}$$

$$\begin{aligned}
& \times \int_0^\infty dt \left\{ \prod_{\bar{i}_c} \Theta_3 \left(\frac{y_1^{\bar{i}_c} - y_2^{\bar{i}_c}}{2\pi R_{\bar{i}_c}} \mid \frac{i\alpha't}{\pi(R_{\bar{i}_c})^2} \right) \right. \\
& \times \left(\sqrt{\frac{\pi}{\alpha't}} \right)^{d_{i_n}} \exp \left(- \frac{1}{4\alpha't} \sum_{\bar{i}_n} (y_1^{\bar{i}_n} - y_2^{\bar{i}_n})^2 \right) \\
& \times \frac{1}{q} \left(\prod_{m=1}^\infty \left[\left(\frac{1 - q^{2m}}{1 + q^{2m-1}} \right)^2 \frac{\det(1 + \mathcal{S}_{(m-1/2)1} \mathcal{S}_{(m-1/2)2}^T q^{2m-1})}{\det(1 - \mathcal{S}_{(m)1} \mathcal{S}_{(m)2}^T q^{2m})} \right] \right. \\
& \left. \left. - \prod_{m=1}^\infty \left[\left(\frac{1 - q^{2m}}{1 - q^{2m-1}} \right)^2 \frac{\det(1 - \mathcal{S}_{(m-1/2)1} \mathcal{S}_{(m-1/2)2}^T q^{2m-1})}{\det(1 - \mathcal{S}_{(m)1} \mathcal{S}_{(m)2}^T q^{2m})} \right] \right] \right) \\
& \times \frac{1}{\sqrt{\det Q \det K_1 \det K_2}} \\
& \times \exp \left[- \frac{1}{4} \left(E^T Q^{-1} E + \sum_{\alpha'_1, \beta'_1} y_2^{\alpha'_1} y_2^{\beta'_1} (K_1^{-1})_{\alpha'_1 \beta'_1} + \sum_{\alpha'_2, \beta'_2} y_1^{\alpha'_2} y_1^{\beta'_2} (K_2^{-1})_{\alpha'_2 \beta'_2} \right) \right] \Big\} \quad (21)
\end{aligned}$$

where $q = e^{-2t}$, and $V_{\bar{u}}$ is common volume of the branes. The set $\{\bar{i}\}$ shows directions perpendicular to both branes except i_0 , $\{\bar{u}\}$ is for the directions along both branes except 0, $\{\alpha'_1\}$ is used for the directions along the Dp_1 -brane and perpendicular to the Dp_2 -brane and $\{\alpha'_2\}$ indicates the directions along the Dp_2 -brane and perpendicular to the Dp_1 -brane. \bar{i}_c and \bar{i}_n are related to the compact and non-compact parts of \bar{i} , respectively. The matrices Q , K_1 , K_2 and the doublet E are defined through their elements as in the following

$$\left\{ \begin{array}{l} Q_{11} = \frac{\alpha't}{2(V_2 - V_1)^2} (1 + V_1^2)(1 - V_2^2) + 2i\alpha'(U_1^{-1})^{00}(1 - V_2^2)^2, \\ Q_{22} = \frac{\alpha't}{2(V_2 - V_1)^2} (1 + V_2^2)(1 - V_1^2) - 2i\alpha'(U_2^{-1})^{00}(1 - V_1^2)^2, \\ Q_{12} = Q_{21} = \frac{\alpha't}{(V_2 - V_1)^2} (1 + V_1^2)(1 + V_2^2)(1 - V_1 V_2), \end{array} \right. \quad (22)$$

$$\left\{ \begin{array}{l} E_1 = \frac{i}{2(V_2 - V_1)} \left[y_2^{i_0} (1 + V_1^2)^2 - y_1^{i_0} (1 + V_1 V_2) \right], \\ E_2 = \frac{i}{2(V_2 - V_1)} \left[y_1^{i_0} (1 + V_2^2)^2 - y_2^{i_0} (1 + V_1 V_2) \right], \end{array} \right. \quad (23)$$

$$\left\{ \begin{array}{l} K_1^{\alpha'_1 \beta'_1} = 4i\alpha' (1 - \frac{1}{2}\delta_{\alpha'_1 \beta'_1}) (U_1^{-1})^{\alpha'_1 \beta'_1} - \alpha't \delta^{\alpha'_1 \beta'_1}, \\ K_1^{\bar{u}\bar{v}} = 4i\alpha' (1 - \frac{1}{2}\delta_{\bar{u}\bar{v}}) (U_1^{-1})^{\bar{u}\bar{v}} - \frac{1}{2}\alpha't \delta^{\bar{u}\bar{v}}, \\ K_1^{\alpha'_1 \bar{u}} = K_1^{\bar{u} \alpha'_1} = 4i\alpha' (U_1^{-1})^{\alpha'_1 \bar{u}}. \end{array} \right. \quad (24)$$

Under the exchanges $1 \rightarrow 2$ and $i \rightarrow -i$ in the elements of the matrix K_1 we receive the elements of the matrix K_2 . As is obvious Q , E and K 's are completely velocity and tachyon dependent.

In the amplitude (21), the theta function comes from the compact part of the set $\{X^{\bar{i}}\}$, while the exponential and its pre-factor in the third line originate from the non-compact

part of it. In fact, the exponential is a damping factor with respect to the distance of the branes. If all directions $\{X^{\bar{i}}\}$ are compact the exponential and its pre-factor disappear. In this case \bar{i}_c takes all values of \bar{i} . In the case that all directions $\{X^{\bar{i}}\}$ are non-compact the Θ_3 -factor is removed, hence \bar{i}_n takes all values of \bar{i} . The next two lines which contain the \mathcal{S} matrix reflect the portion of the oscillators, conformal ghosts and super conformal ghosts. The remaining part, which is obtained by integration over the momenta, the Eq. (10), is due to the presence of the velocities and the background tachyon fields. In absence of the velocities and tachyon fields, this factor is absent too and hence the interaction amplitude resembles to the one in [3].

3.2 Interaction amplitude in the R-R sector

For interaction amplitude in the R-R sector we use the total GSO projected boundary state for the R-R sector, the Eq. (41), and follow the same procedure in the NS-NS sector, so

$$\begin{aligned}
\mathcal{A}_{\text{R-R}} = & \frac{\alpha' V_{\bar{u}}}{8(2\pi)^{d_{\bar{i}}}} \frac{T_{p_1} T_{p_2}}{|V_1 - V_2|} \int_0^\infty dt \left\{ \prod_{\bar{i}_c} \Theta_3 \left(\frac{y_1^{\bar{i}_c} - y_2^{\bar{i}_c}}{2\pi R_{\bar{i}_c}} \mid \frac{i\alpha' t}{\pi(R_{\bar{i}_c})^2} \right) \right. \\
& \times \left(\sqrt{\frac{\pi}{\alpha' t}} \right)^{d_{\bar{i}_n}} \exp \left(-\frac{1}{4\alpha' t} \sum_{\bar{i}_n} (y_1^{\bar{i}_n} - y_2^{\bar{i}_n})^2 \right) \\
& \times \left(\zeta \prod_{m=1}^\infty \left[\left(\frac{1 - q^{2m}}{1 + q^{2m}} \right)^2 \frac{\det(1 + \mathcal{S}_{(m)1} \mathcal{S}_{(m)2}^T q^{2m})}{\det(1 - \mathcal{S}_{(m)1} \mathcal{S}_{(m)2}^T q^{2m})} \right] + \zeta' \right) \\
& \times \frac{1}{\sqrt{\det Q \det K_1 \det K_2}} \\
& \times \exp \left[-\frac{1}{4} \left(E^T Q^{-1} E + \sum_{\alpha'_1, \beta'_1} y_2^{\alpha'_1} y_2^{\beta'_1} (K_1^{-1})_{\alpha'_1 \beta'_1} + \sum_{\alpha'_2, \beta'_2} y_1^{\alpha'_2} y_1^{\beta'_2} (K_2^{-1})_{\alpha'_2 \beta'_2} \right) \right] \Big\} \quad (25)
\end{aligned}$$

where

$$\zeta \equiv -\frac{1}{2} \text{Tr}[G_1 C^{-1} G_2^T C], \quad (26)$$

$$\zeta' \equiv -i \text{Tr}[G_1 C^{-1} G_2^T C \Gamma_{11}], \quad (27)$$

and $G_{1,2} = \exp[\frac{1}{2}(\Phi_{(1,2)})_{\mu\nu} \Gamma^\mu \Gamma^\nu]$. Note that the variables ζ and ζ' implicitly depend on the branes' dimensions through Φ_1 and Φ_2 in G_1 and G_2 .

Now we are eager to study the total amplitude, i.e. the combination of the amplitudes in the NS-NS and R-R sectors. Consider the following special case: there is no compactification, the two Dp-branes are parallel with the same dimensions, and the same fields living on them. Thus, as in the literature, this interaction amplitude becomes zero due to the cancellation of attractive and repulsive forces in the NS-NS and R-R sectors, respectively.

In the case at hand, beside the living fields on the branes, the velocities which are transverse fluctuations of the branes are present, too. In the amplitude (21) and (25) the relative speed appeared in the denominators. This puts a constraint on the system that the velocities of the branes should be different, otherwise the total amplitude becomes infinite. In this case, we can not check the vanishing of the interaction amplitude for identical parallel branes with the same fields. Therefore, even if all the fields are identical, the velocities should be different. This causes the branes to have different Φ 's and consequently different \mathcal{S} 's. So the NS-NS and R-R amplitudes cannot cancel the effect of each other.

4 Long distance behavior of the amplitude

Now we find the interaction between the branes when they are far from each other. That is, we find the behavior of the interaction amplitudes (21) and (25) when time goes to infinity. Conventionally, in the large distance only the massless states of the closed string contribute to the branes interaction.

The large distance amplitude is equivalent to the long time behavior of the branes. It can be acquired by sending q to zero in the Eqs. (21) and (25). So the interaction amplitudes due to massless states in the NS-NS and R-R sectors are

$$\begin{aligned} \lim_{q \rightarrow 0} \mathcal{A}_{\text{NS-NS}} = & \frac{V_{\bar{u}} T_{p_1} T_{p_2}}{4(2\pi)^{d_{\bar{t}}}} \frac{i(-1)^{(p_1+p_2)/2} 2^{d_{\bar{u}}+1/2}}{\alpha'^{(p_1+p_2)/2} (1+V_1^2)(1+V_2^2)} \prod_{m=1}^{\infty} \frac{\det[M_{(m-1/2)1} M_{(m-1/2)2}]}{\det[M_{(m)1} M_{(m)2}]} \\ & \times \int^{\infty} dt \left\{ \left(\sqrt{\frac{\pi}{\alpha' t}} \right)^{d_{\bar{t}_n}} \exp \left(-\frac{1}{4\alpha' t} \sum_{\bar{t}_n} (y_1^{\bar{t}_n} - y_2^{\bar{t}_n})^2 \right) \right. \\ & \left. \times \lim_{t \rightarrow \infty} \frac{2[\text{Tr}(\mathcal{S}_{(1)1} \mathcal{S}_{(1)2}^T) - 2]}{t^{1+(p_1+p_2)/2}} \right\}, \end{aligned} \quad (28)$$

and

$$\begin{aligned} \lim_{q \rightarrow 0} \mathcal{A}_{\text{R-R}} = & \frac{V_{\bar{u}} T_{p_1} T_{p_2}}{8(2\pi)^{d_{\bar{t}}}} \frac{i(-1)^{(p_1+p_2)/2} 2^{d_{\bar{u}}+1/2}}{\alpha'^{(p_1+p_2)/2} (1+V_1^2)(1+V_2^2)} \\ & \times \int^{\infty} dt \left\{ \left(\sqrt{\frac{\pi}{\alpha' t}} \right)^{d_{\bar{t}_n}} \exp \left(-\frac{1}{4\alpha' t} \sum_{\bar{t}_n} (y_1^{\bar{t}_n} - y_2^{\bar{t}_n})^2 \right) \right. \\ & \left. \times \lim_{t \rightarrow \infty} \frac{1}{t^{1+(p_1+p_2)/2}} \right\}. \end{aligned} \quad (29)$$

We do not put the limit on the exponential part and its pre-factor in the Eqs. (28) and (29) because these factors are related to the positions of the branes, and closed string emission is independent of the locations of the branes. When there is no tachyonic background [3], last factors in the Eqs. (28) and (29) do not have the factor $1/t^{1+(p_1+p_2)/2}$. Thus, due to

the presence of tachyon fields, the interaction amplitude decreases in time. In fact, the statement that for large distances of the branes the massless closed string states dominate in the interaction is valid until there is no tachyon backgrounds in the system.

There is an interpretation for this unusual behavior. In fact, the open string tachyon background causes an instability in the system. Therefore, after long enough time, by rolling of the tachyon [12] toward its minimum potential, unstable D-branes decay to the bulk modes and their dimensions decrease to reach a stable system. Final products of this process are branes with lower dimensions or vacuum of the closed string [10]. The latter implies that there are no physical perturbative open string states around the minimum of the potential. This is due to the fact that the open string states live only on the branes. Thus, in the concept of interactive branes, by passing the time which leads to tachyon rolling and decreasing of their dimensions, the branes' configuration distorts and prevents them from the interaction.

The amplitude $\mathcal{A}_{\text{NS-NS}}$ in the Eq. (28) depends on the background fields through the factor $\text{Tr}(\mathcal{S}_{(1)1}\mathcal{S}_{(1)2}^T)$ and the determinants of the matrices $\{M_{(m-1/2)}|m = 1, 2, 3, \dots\}$, while such a dependence is absent in the amplitude $\mathcal{A}_{\text{R-R}}$, the Eq. (29). In other words, when the branes are far from each other, the R-R amplitude becomes background independent.

Another interesting feature of the long time amplitude is its time-dependent behavior on the branes' dimensions. An exception here is the D-instanton. When two D-instantons, which have the dimension $p_1 = p_2 = -1$, interact the factor $1/t^{1+(p_1+p_2)/2}$ is removed and the amplitude behavior in long time is resembled to a system without tachyon. For this system the presence of the tachyon does not affect the conventional behavior of the large distance interaction.

5 Conclusions

The boundary state of a closed superstring traveling between two moving branes in the presence of $B_{\mu\nu}$, tachyon and $U(1)$ gauge field was calculated. Notable feature in the boundary state equations is the prevention of the closed string wrapping around the compact directions of spacetime, which is due to the presence of the tachyon field. As well, the boundary state includes a momentum dependent exponential factor which is absent in the conventional boundary states. This factor originates from the zero mode parts of the velocity and tachyon terms in the boundary action.

The interaction amplitude of the branes via exchanging of closed string was calculated for the NS-NS and R-R sectors. It is shown that even for co-dimension parallel branes with

similar external fields, the total amplitude is not zero. This is due to the presence of the velocities and tachyon fields in the system.

The long distance behavior of the interaction amplitude was studied. In this domain the instability of the branes, due to the background tachyon fields, weakens the interaction. This decreasing behavior can be understood by dissipation of the branes to the bulk modes because of the rolling of the tachyon to its minimum potential in long time regime. The interaction for two D-instantons obviates this decreasing behavior. Thus, the long time amplitude in this case behaves like the conventional case in which the massless states dominate.

Appendix A

Zero mode boundary state in the R-R sector

The state $|B_\psi, \eta\rangle_{\text{R}}^{(0)}$ in the Eq. (16) is the zero mode boundary state that obeys the following equation

$$|B_\psi, \eta\rangle_{\text{R}}^{(0)} = \mathcal{M}^{(\eta)AB} |A\rangle |\tilde{B}\rangle, \quad (30)$$

where $|A\rangle |\tilde{B}\rangle$ is the vacuum of the zero modes d_0^μ and \tilde{d}_0^μ . The matrix $\mathcal{M}^{(\eta)}$ has to satisfy the equation

$$(\Gamma^\mu)^T \mathcal{M}^{(\eta)} - i\eta \mathcal{S}_{(m)\nu}^\mu \Gamma_{11} \mathcal{M}^{(\eta)} \Gamma^\nu = 0. \quad (31)$$

Consider a solution in the form

$$\mathcal{M}^{(\eta)} = C \Gamma_{11} \left(\frac{1 + i\eta \Gamma_{11}}{1 + i\eta} \right) G, \quad (32)$$

in which C is the charge conjugate matrix. Substitution of the Eq. (32) into the Eq. (31) leads to the following equation for the matrix G ,

$$\Gamma^\mu G = \mathcal{S}^\mu{}_\nu G \Gamma^\nu. \quad (33)$$

There is a conventional solution for G as

$$G = \exp\left(\frac{1}{2} \Phi_{\mu\nu} \Gamma^\mu \Gamma^\nu\right). \quad (34)$$

Indeed one must expand the exponential with the convention that all gamma matrices anticommute, therefore there are a finite number of terms. The antisymmetric matrix Φ is defined in terms of the matrix \mathcal{S} ,

$$\mathcal{S} = (1 - \Phi)^{-1} (1 + \Phi). \quad (35)$$

Appendix B

GSO projected and ghosts boundary states

The GSO projected boundary states are given by

$$|B, \tau = 0\rangle_{\text{NS}} = \frac{1 - (-1)^{F+G}}{2} \frac{1 - (-1)^{\tilde{F}+\tilde{G}}}{2} |B, \eta = +1, \tau = 0\rangle_{\text{NS}}, \quad (36)$$

$$|B, \tau = 0\rangle_{\text{R}} = \frac{1 + (-1)^n (-1)^{F+G}}{2} \frac{1 - (-1)^{\tilde{F}+\tilde{G}}}{2} |B, \eta = +1, \tau = 0\rangle_{\text{R}}, \quad (37)$$

where n is an even number for the type IIA superstring theory and is odd for the IIB superstring theory. The definitions of F and G , are

$$F = \sum_{r=1/2}^{\infty} b_{-r}^{\mu} b_{r\mu}, \quad G = - \sum_{r=1/2}^{\infty} (\gamma_{-r} \beta_r + \beta_{-r} \gamma_r), \quad (38)$$

for the NS-NS sector, and

$$(-1)^F = \Gamma_{11} (-1)^{\sum_{m=1}^{\infty} d_{-m}^{\mu} d_{m\mu}}, \quad G = -\gamma_0 \beta_0 - \sum_{m=1}^{\infty} (\gamma_{-m} \beta_m + \beta_{-m} \gamma_m), \quad (39)$$

for the R-R sector. Similar definitions also hold for \tilde{F} and \tilde{G} . Thus, the total projected boundary states are

$$|B, \tau = 0\rangle_{\text{NS}} = \frac{1}{2} \left(|B, +, \tau = 0\rangle_{\text{NS}} - |B, -, \tau = 0\rangle_{\text{NS}} \right), \quad (40)$$

$$|B, \tau = 0\rangle_{\text{R}} = \frac{1}{2} \left(|B, +, \tau = 0\rangle_{\text{R}} + |B, -, \tau = 0\rangle_{\text{R}} \right). \quad (41)$$

Since the bulk action in the Eq. (1) preserves conformal symmetry, working in covariant formalism necessitates including conformal ghosts [2, 16]. In fact, what we need is the portion of ghosts (i.e. anti-commuting fields b , c , \tilde{b} and \tilde{c}) in the bosonic boundary state. This part is independent of the background fields and is expressed as

$$|B_{\text{gh}}, \tau = 0\rangle = \exp \left[\sum_{m=1}^{\infty} e^{4im\tau_0} (c_{-m} \tilde{b}_{-m} - b_{-m} \tilde{c}_{-m}) \right] \frac{c_0 + \tilde{c}_0}{2} |q = 1\rangle |\tilde{q} = 1\rangle. \quad (42)$$

In the superstring theory, in addition to the conformal ghosts, we should also consider the super conformal ghosts. Thus, the boundary state, corresponding to the super conformal ghosts in the NS-NS and R-R sectors, are as in the following

$$|B_{\text{sgh}}, \eta, \tau = 0\rangle_{\text{NS}} = \exp \left[i\eta \sum_{r=1/2}^{\infty} (\gamma_{-r} \tilde{\beta}_{-r} - \beta_{-r} \tilde{\gamma}_{-r}) \right] |P = -1\rangle |\tilde{P} = -1\rangle. \quad (43)$$

$$|B_{\text{sgh}}, \eta, \tau = 0\rangle_{\text{R}} = \exp \left[i\eta \sum_{m=1}^{\infty} (\gamma_{-m} \tilde{\beta}_{-m} - \beta_{-m} \tilde{\gamma}_{-m}) \right] |P = -\frac{1}{2}\rangle |\tilde{P} = -\frac{3}{2}\rangle. \quad (44)$$

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